Anelastic benchmark suggestions (Glatzmaier)

Choose an ideal gas and construct the reference state (or the initial mean state) that is adiabatic and hydrostatic. Maybe choose to update the reference state periodically so it better approximates the evolving mean state.

Maybe constant $\bar{\nu}$, $\bar{\kappa}$, $\bar{\eta}$, μ, c_P, γ. (Although a radially-dependent $\bar{\eta}$ would be a better test.)

Boundary conditions: stress-free, impermeable, constant entropy at inner boundary, constant heat flux at outer boundary, magnetic field mapped to a potential field at outer boundary, and the core below the convection zone could have either a finite or infinite conductivity (the former would require solving for B throughout the interior).

$$\bar{p} = R\bar{\rho}\overline{T} \qquad \frac{d\bar{p}}{dr} = -\bar{\rho}\bar{g}$$

$$\frac{d\overline{S}}{dr} = 0 \qquad \frac{d\overline{T}}{dr} = -\bar{g}/c_P$$

$$\boldsymbol{\nabla}\cdot\bar{\rho}\mathbf{v} = 0$$

$$\boldsymbol{\nabla}\cdot\mathbf{B} = 0$$

$$\rho' = -\frac{\bar{\rho}}{c_P}S' + \frac{\bar{\rho}}{\gamma\bar{p}}p'$$

$$\bar{\rho}\frac{d\mathbf{v}}{dt} = -\boldsymbol{\nabla}p' - \bar{g}\rho'\hat{\mathbf{r}} + 2\bar{\rho}\mathbf{v}\times\boldsymbol{\Omega} + \boldsymbol{\nabla}\cdot(2\bar{\rho}\bar{\nu}(e_{ij} - \frac{1}{3}\boldsymbol{\nabla}\cdot\mathbf{v}\delta_{ij}) + \frac{1}{\mu}(\boldsymbol{\nabla}\times\mathbf{B})\times\mathbf{B}$$

Where e_{ij} is the rate of strain tensor.

$$\bar{\rho}\overline{T}\frac{dS'}{dt} = \boldsymbol{\nabla}\cdot(\overline{T}\bar{\rho}\bar{\kappa}\boldsymbol{\nabla}S') + 2\bar{\nu}\bar{\rho}(e_{ij}e_{ij} - \frac{1}{3}(\boldsymbol{\nabla}\cdot\mathbf{v})^2) + \frac{\bar{\eta}}{\mu}|\boldsymbol{\nabla}\times\mathbf{B}|^2$$

$$\frac{\partial\mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{B}) - \boldsymbol{\nabla}\times(\eta\boldsymbol{\nabla}\times\mathbf{B})$$